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CLUMPY COLD DARK MATTER

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ABSTRACT

Clumpiness is likely to be generic to cold dark matter cosmologies. We consider cold dark matter models with cosmic strings and textures appropriate for galaxy formation. CDM clumps with a density of $10^7 \mathrm{M}_\odot \mathrm{pc}^{-3}$ are generated at redshift $\sim z_{\rm eq}$ and a sizable fraction of them may survive to today. The most numerous clumps should have dense cores of mass $\sim 10^{-2}(1\,\mathrm{GeV}/M_x)^{\frac{3}{2}}\,\mathrm{M}_\odot$, where M_x is the mass of the dark matter particle, and might contain up to 10^{-3} of the CDM mass. Even in canonical, unseeded cold dark matter models, there is also likely to be clumpiness, developing when the first rare fluctuations go non-linear, and surviving on scales of up to $10^6 \mathrm{M}_\odot$ in galaxy halos. Observable implications include possible dark matter cores in globular clusters, and in galactic nuclei. The enhanced annihilation rate in clumps can lead to a significant contribution to the diffuse γ-ray background, as well as emission from the Galactic center. Results from terrestrial dark matter detection experiments might be significantly affected by clumpiness in the Galactic halo.

I. INTRODUCTION

Cold dark matter provides a generic solution to the dark matter problem. Alternative candidates abound for stable weakly interacting relic particles from the early universe. The favored mass scale for these particles is at or above the electro-weak unification scale ($\sim$ 100 GeV), since any new symmetries which require the introduction of new, exotic, and possibly stable, particles, such as SUSY, should only be dominant above this scale. Relic massive particles were initially in thermal equilibrium, and freeze-out estimates of their abundances, combined with direct searches for particles and their annihilation signatures, constrain their masses phenomenologically to exceed $\sim$ 1GeV. In fact, for dark matter particles to be cold ($v << c$) at the epoch when the universe was first matter-dominated, at a redshift $1+z = 4\times 10^4\, \Omega h^2$, we only require $M_x > \mathcal{O}(1)$ keV (Bond & Szalay 1983).

Non-baryonic dark matter is also motivated by inflation, which requires that $\Omega = 1$ in contrast to the standard primordial nucleosynthesis constraint (Olive *et al.* 1991) that $\Omega_b h^2 \approx 0.02$. A lower bound on the density of non-baryonic dark matter may plausibly be imposed by the observed dark matter in halos, clusters and superclusters of galaxies, which amounts to $\Omega \sim 0.15$ with an uncertainty of about a factor of 2. Combining all of the previous considerations, we conclude that a reasonable working hypothesis is accordingly that the prevalent dark matter is non-baryonic, cold and lies in the range bounded by

$$0.1 \lesssim \Omega_{cdm} \lesssim 1. \tag{1.1}$$

Cold dark matter in the context of inflation has received considerable support for its success in accounting for many features of large-scale structure in a $\Omega = 1$ universe. The inflationary prediction of Gaussian, scale-invariant curvature fluctuations leads to the generation of large-scale structure via gravitational instability and growth of linear perturbations. Successes include the simultaneous explanation of galaxy–galaxy correlations on small scales, halo abundances and rotation curves, galaxy peculiar velocities, and large–scale voids, filaments and superclusters (e.g., Frenk *et al.* 1988). However there are also notable failures, including the angular correlations of galaxies to $z \sim 0.2$ (Maddox *et al.* 1990) and the variance in the cell counts of galaxies (Efstathiou *et al.* 1990).

Consequently, rival theories of large-scale structure formation continue to merit attention. One class of such theories involves the formation of topological defects in an early phase transition. These relics of high density interact dynamically on horizon scales throughout the ensuing history of the universe, and provide characteristically non-linear, non-Gaussian seeds for large-scale structure. Examples of such non-linear seeds include cosmic strings (Vilenkin 1985) and textures (Turok 1989). Of these possibilities, cosmic strings and textures have been simulated in sufficient detail to be able to develop a quantitative model for galaxy and cluster formation.

We shall henceforth assume that the universe is dominated by cold dark matter. Our aim is to study the degree to which some of the cold dark matter may retain very high densities in the context of three well-developed scenarios for cosmic structure formation, fluctuations induced by cosmic strings (§II), textures (§III), and inflation (§IV). In the first two cases, one may have to dispense with inflation, and $\Omega_{\rm cdm}$ is then bounded by (1); in the latter case $\Omega_{\rm cdm} = 1$. In §V we discuss the survivability of clumps. The observable implication of CDM clumps is discussed in the final section.

II. COSMIC STRINGS

In models with CDM and a sufficient amplitude of density inhomogeneity on small scales, nonlinear structures will form before matter-radiation equality. This is true of models with Gaussian perturbations with a power law spectrum of $n \gtrsim -2$ on the smallest scales or for models where the structure is seeded by cosmic strings. In this section we will look at cosmic string models and assuming that the string mass parameter takes the correct value so as to seed the observed homogeneities. Thus the linear mass density of strings, μ, satisfies $G\mu/c^2 \sim 10^{-6}$. In Gaussian models the nonlinearities will be distributed roughly homogeneously throughout the universe, with the density at a typical point deviating significantly from the mean. However for inhomogeneities which are created by cosmic strings, the nonlinearities will be isolated. The density at most points will be quite near the mean but the density will deviate significantly from the mean in isolated regions. Since self-gravity of the CDM is generally unimportant in the radiation era, the nonlinearities will not virialize. Thus one will not obtain the large density contrasts found for virialized objects. Typically the density will remain a few times greater or smaller than the mean, the particles just moving ballistically in the expanding universe.

Given that the overdensities do not grow many times greater than the mean density, one must wait until $z_{\rm eq}$ for these nonlinear structures to collapse and virialize. Once the structures have collapsed in all three dimensions, their density will no longer shrink with the expansion but will remain constant. Secondary infall of CDM onto these clumps will cloak these objects in virialized halos of smaller density. However for the purposes of this paper, the central dense cores are the most interesting. Since the collapse occurs at $z \sim z_{\rm eq}$, the density of the cores is approximately the density of the universe at $z_{\rm eq}$ or

$$\rho_{\rm core} \sim \rho_{\rm eq} = 8.7 \times 10^6\, h^8 \Omega_0^4\, {\rm M}_\odot\, {\rm pc}^{-3}, \tag{2.1}$$

assuming that CDM is the dominant form of matter besides photons and neutrinos.

Let us first consider the perturbation immediately around a moving piece of string. The primary effect of a passing piece of string is to give a velocity boost to the matter surrounding it. The comoving distance that a CDM particle moves before $z_{\rm eq}$, given an initial peculiar velocity v_s at time $t_{\rm i}$, is

$$\Delta = \frac{v_s\, t_{\rm i}}{a_{\rm i}} \ln \frac{t_{\rm eq}}{t_{\rm i}} \qquad v_s = \epsilon_s c. \tag{2.2}$$

Immediately surrounding a moving segment of string, the size of the velocity boost is $4\pi G\mu\beta\gamma = \epsilon_s c$, where β and γ are the usual relativistic velocity factors of the string.

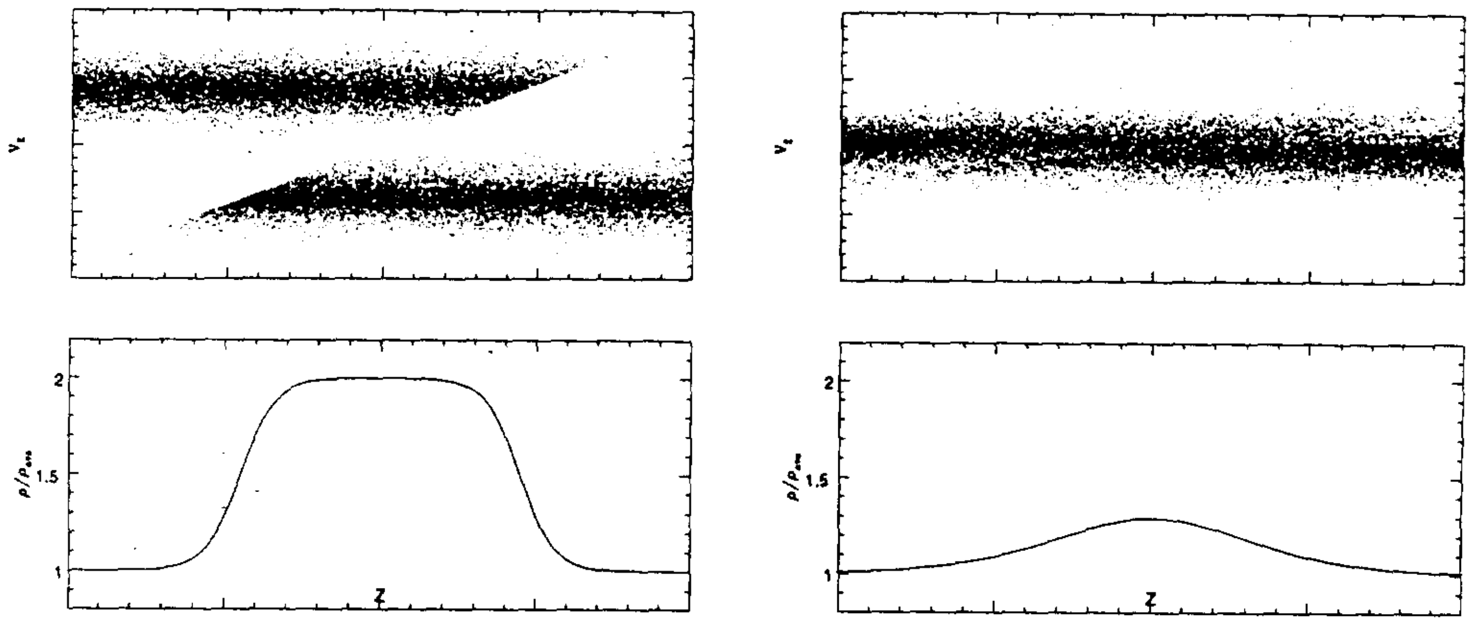

FIGURE 1 Shown is the phase space distribution of particles in the z direction after a segment of string passed through the x - y plane. Below is plotted the density of particles for two cases; a) the velocity boost given to the particles by the passing string is greater than the velocity dispersion and produces an overdensity of two, and b) the velocity boost is smaller than the dispersion and a smaller overdensity results.

Typically $\beta\gamma \approx 1$, and so $\epsilon_s \sim 10^{-5}$. The direction of the velocity boost is toward the surface swept out by the string. The two streams of particles moving toward each other will start to overlap. If the boost is greater than the velocity dispersion of the particles, then in the overlapping region the two streams of particles will cause a density enhancement of a factor of two as illustrated in figure 1a. However, if the velocity boost is less than the velocity dispersion, the overdensity will become small as illustrated in figure 1b. In either case, the overdensity will not grow in the radiation era but the region of overlap and hence the region of overdensity will grow logarithmically with time. The total thickness of the wake at t_{eq} is just 2Δ. At z_{eq} the nonlinear wakes (i.e., figure 1a) will collapse while the linear wakes (figure 1b) will have to wait for some matter era growth before they can collapse. We are interested in the densest bound objects, and so we will ignore the linear wakes.

Which wakes will be linear and which nonlinear? Since the velocity boost given by a passing string is independent of epoch and the velocity dispersion of the CDM will decay due to the expansion of the universe, only wakes produced after some critical redshift, z_{damp}, will produce nonlinear wakes. Since the size of the wakes is an increasing function of the time at which they are produced, this will correspond to a lower cutoff to the size of nonlinear wakes due to CDM free-streaming.

In general the velocity dispersion of the CDM is given by

$$v_{\mathrm{cdm}}(z) \sim \sqrt{3}\, g_{\mathrm{dec}}^{-\frac{1}{3}} \frac{T_0}{\sqrt{M_x T_{\mathrm{dec}}}} z \sim 10^{-11}\, c \left(\frac{11}{4g_{\mathrm{dec}}}\right)^{\frac{1}{3}} \left(\frac{1\,\mathrm{GeV}}{M_x}\right)^{\frac{1}{2}} \left(\frac{1\,\mathrm{MeV}}{T_{\mathrm{dec}}}\right)^{\frac{1}{2}}, \qquad (2.3)$$

where the redshift at decoupling is $z_{\mathrm{dec}} = g_{\mathrm{dec}}^{\frac{1}{3}} T_{\mathrm{dec}}/T_0$, T_0 is the present microwave temperature $\sim 2.7\,\mathrm{K}$, and g_{dec} is the effective fractional increase in relativistic degrees of freedom at z_{dec} compared to today. The condition that the string perturbations are not damped by free-streaming is $v_{\mathrm{cdm}} < \epsilon_s c$. Hence the redshift prior to which damping in unimportant is

$$z_{\mathrm{damp}} = \frac{\epsilon_s c}{v_{\mathrm{cdm}}(0)} \sim 5 \times 10^5 \left(\frac{4g_{\mathrm{dec}}}{11}\right)^{\frac{1}{3}} \left(\frac{M_x}{1\,\mathrm{GeV}}\right)^{\frac{1}{2}} \left(\frac{T_{\mathrm{dec}}}{1\,\mathrm{MeV}}\right)^{\frac{1}{2}}. \qquad (2.4)$$

The preceding analysis assumes that the curvature of the string is smaller than both the free-streaming length at production and the width of the wakes. The smallest loops around at time t_i are the one which are just about to lose all of their mass to gravitational radiation and have a length $\sim 100\, G\mu t_i/c \sim 10^{-4} ct_i$ and a typical curvature of roughly $\frac{1}{10}$ of their length. These small loops, in fact, have most of the mass. The width of a wake produced by these loops at t_i is is roughly $\epsilon_s \ln(t_{\mathrm{eq}}/t_i)/ \sim 10^{-5} ct_i \ln(t_{\mathrm{eq}}/t_i)$, so the smallest loops will produce wakes which are about the size of the loop or somewhat bigger for any value of $G\mu$. Thus the nonlinear overdensities formed by these wakes will not be the simple planar structures described above, but will be considerably more complicated. Nevertheless the amount of nonlinear CDM produced by a length of relativistic string will be roughly the same as whether that string is part of a small loop or part of a straight string. The effect of the more complicated geometries would be to make the nonlinear regions overlap without changing the total mass involved. This would lead to overdensities larger than 2 and thus collapse somewhat before z_{eq} and formation of denser virialized objects.

To estimate the total mass contained in these nonlinear overdensities, note that the nonlinear mass produced by a piece of string is given by

$$\frac{d\delta M_{\mathrm{nl}}}{dt_i} = \bar{\rho}_{\mathrm{cdm}}\, a_i \Delta \frac{dA}{dt_i} \qquad a\Delta \sim 2\epsilon_s ct_i \ln\frac{t_{\mathrm{eq}}}{t_i} \qquad \frac{dA}{dt_i} \sim \beta c\delta L \qquad (2.5)$$

where Δ/a is the thickness of the wake at z_{eq} scaled to physical units at time t_i, and A is the physical area swept out by the string. Now the mass of the segment of string is just $\gamma\mu\delta L$. Averaging over all of space we find the density of nonlinear mass produced at a given time:

$$\frac{d\rho_{\mathrm{nl}}}{dt_i} \sim 2\epsilon_s \bar{\beta} G \bar{\rho}_{\mathrm{cdm}} \bar{\rho}_s ct_i \ln\frac{t_{\mathrm{eq}}}{t_i} \qquad \bar{\beta} \approx \frac{1}{\sqrt{2}} \qquad (2.6)$$

where $\bar{\rho}_s$ is the total density of strings at time t_i. Using $\bar{\rho} = \frac{3}{32\pi G t_i^2}$ we may calculate the fraction of the CDM contained in the nonlinear structures at z_{eq} which were seeded at a given epoch:

$$\frac{df_{nl,eq}}{d\ln z_i} \sim \frac{3}{2}\Omega_s \ln\frac{z_i}{z_{eq}} \qquad \Omega_s \equiv \frac{\bar{\rho}_s}{\bar{\rho}}. \tag{2.7}$$

The fraction of the cosmic density in strings, Ω_s, will be constant in the radiation era, given by the scaling solution. We may thus integrate this equation for z_{eq} to z_{damp} finding the total fraction of CDM which is nonlinear at z_{eq}

$$f_{nl,eq} \sim \Omega_s \left[\ln\frac{z_{damp}}{z_{eq}}\right]^2. \tag{2.8}$$

The density in small loops which are just about to decay via gravitational radiation should dominate, giving the scaling relation $\Omega_s \propto \sqrt{G\mu/c^2}$. The coefficient in front is somewhat uncertain, depending on the distribution of loops produced, the amount of loop fragmentation, and effects due to the back-reaction of the gravitational radiation on the loops. However Ω_s is probably in the range $10^{-2} - 10^{-3}$. The logarithmic term in (2.8) would give a factor of ~ 10 for WIMPs. Other candidates for dark matter may give a somewhat different value. We thus obtain for the fraction of CDM particles which virialize at z_{eq}:

$$f_{nl,eq} \sim 0.1 \tag{2.9}$$

We must require that an object collapse in all three directions before the density becomes constant, being unaffected by the universal expansion. This is clearly violated by the planar wakes discussed above. However as mentioned above most of the nonlinear structures are caused by the smallest loops with sizes comparable to the thickness of the wakes. These loops will be moving rapidly due to the thrust from the gravitational radiation. Numerical determinations of the gravitational radiation from loops have found that the typical ratio of momentum emitted in gravity waves to power emitted in gravity waves is $\dot{p}/\dot{E} = 0.1/c$. Thus we expect that the loops which have lost half their mass will be moving at ~ 0.1c. The period of the loop of mass M is $0.5M/(\mu c)$, so the distance traveled in one oscillation is $\sim 0.05M/\mu$. Now the typical radius of a loop of mass M will, of course, depend on its shape, but for many simple non-intersecting loop trajectories it is $\sim 0.1M/\mu$. Thus we see that the wakes left by these smallest loops will corrugated by the loop oscillations, and the typical distance between corrugations will be about the same as the transverse size of the accretion wakes. Furthermore, as shown above, the nonlinear region around the wake will be roughly the same size as this transverse dimension. The correspondence of these three length scales is not a lucky coincidence, rather it is a consequence of the way these quantities scale with μ. They would all be the same, no matter what the value of μ was. The significance of the coincidence between the loop size and the wake thickness is that the accretions wakes will collapse in two dimensions rather than one. The collapse will be along a curve given by the loop trajectory. The significance of the coincidence of the corrugation length to the transverse size is that these corrugations will cause the accretion wakes to fragment into small cloudlets once the CDM becomes self-gravitating at $z_{\rm eq}$. Hence the accretion wake will collapse in all three dimensions at $z_{\rm eq}$ and the value of $\rho_{\rm eq}$ will properly represent their density.

Now let us consider the size, mass, and internal velocity distribution of these clump cores. Their physical size is roughly the comoving size of the wake thickness scaled to $z_{\rm eq}$ or

$$R_{\rm core} \sim \epsilon_s c t_{\rm i} \sqrt{\frac{t_{\rm eq}}{t_{\rm i}} \ln \frac{t_{\rm eq}}{t_{\rm i}}} \in 0.01{\rm pc} \left[1, \frac{z_{\rm eq}}{z_{\rm damp}} \ln \frac{z_{\rm damp}}{z_{\rm eq}}\right] \Omega_0^{-2} h^{-4}, \tag{2.10}$$

so their masses are in the range

$$M_{\rm core} = \rho_{\rm eq} R_{\rm core}^3 \in 30\,{\rm M}_\odot \left[1, \left(\frac{z_{\rm eq}}{z_{\rm damp}} \ln \frac{z_{\rm damp}}{z_{\rm eq}}\right)^3\right] \Omega_0^{-2} h^{-4} \tag{2.11}$$

and their internal velocity dispersions are in the range

$$v_{\rm core} = \sqrt{\frac{G M_{\rm core}}{R_{\rm core}}} \in 3\,{\rm km\,s^{-1}} \left[1, \left(\frac{z_{\rm eq}}{z_{\rm damp}} \ln \frac{z_{\rm damp}}{z_{\rm eq}}\right)^2\right]. \tag{2.12}$$

The distribution of masses of clump cores is roughly

$$n(M_{\rm core})\, dM_{\rm core} \underset{\sim}{\propto} \left[\ln \frac{30{\rm M}_\odot}{M_{\rm core}}\right]^3 \frac{dM_{\rm core}}{M_{\rm core}^2}, \tag{2.13}$$

so that each mass scale has almost the same amount of mass in it but with a logarithmic weighting toward the low mass end.

The cloudlets we have just considered will become centers of accretion of more CDM. Hence we have referred to these cloudlets as cores. These cores will accrete halos. These cloudlets are just fragments of the accretion wakes of the strings so they will necessarily be near neighboring clumps which will compete for the surrounding CDM. Even if the clumps were isolated they could at most accrete mass proportionally to the redshift after $z_{\rm eq}$ leading to a $\rho \sim r^{-\frac{9}{4}}$ halo. The large fraction of mass already in halos (equation 2.9) suggests that competition will limit the average smooth halo to be ~ 10 times the core mass (accretion of other much smaller clumps may lead to clumpy halos). Competition will also lead to halos with a steeper density profile.

It is important to consider the clumpiness of the CDM medium today, i.e., the mean square density. Given the steep density profile of the halos of the clumps it is clear that the clumpiness will be dominated by the cores of the clumps. Let us suppose that a fraction $f_{\rm cl}$ of the clumps survive until today retaining their initial density. The clumpiness of CDM today is then

$$C \equiv \overline{\delta_{\rm cdm}^2} \sim f_{\rm nl} f_{\rm cl} {z_{\rm eq}}^3 \sim 10^{12} f_{\rm cl} h^6 \Omega_0^3. \tag{2.14}$$

We consider the appropriate value of $f_{\rm cl}$ in §V below.

III. TEXTURES

Another theory in which very large overdensities are produced at early times is the theory of cosmic textures. Turok (1989) has shown that although textures do not have any isolated defects, such as strings or monopoles, they do have a a topological charge or knottedness distributed throughout space. As the cosmological horizon grows the amount of charge within the horizon will also grow just because of $\sqrt{N}$ fluctuations. Once the knottedness within the horizon becomes great enough, the knot will collapse to a point and unwind by temporarily leaving the vacuum manifold. These unwinding points will lead to early formation of bound objects (Gooding, Spergel, and Turok 1991) and must inevitably generate substructure on all scales of interest.

Gooding *et al.* (1991) used the turnaround radius calculated via linear theory as an indicator of the final virialized radius of a given shell of CDM. However, this is valid in the matter era but not necessarily in the radiation era. As we have seen for the planar perturbations produced by cosmic strings, CDM may turnaround and then just keep going out the other side. This is because in linear theory in the radiation era the inertia of the CDM dominates, the self-gravity of the CDM being totally negligible. For the planar geometry of a cosmic string wake, the maximum overdensity of 2 is reached. However for perfectly spherical accretion, the geometry will cause the density to diverge in the center even ignoring the CDM self-gravity. It is then not so clear that nonlinear effects may allow the CDM to virialize in the radiation era. This is a crucial issue to us since a central density much larger than the density at z_{eq} may result, leading to a much larger clumpiness factors.

To address the issue of radiation-era virialization, let us first assume no such virialization and see if this is self-consistent. First let us note that the claim is that collapsing textures become more spherical as they collapse (Turok and Spergel 1990). If this is so then the central regions of the induced perturbation should be very spherical indeed. Since the texture knot collapse approaches the speed of light there is no sense in which the collapsing knot can be moving, the collapse being just as spherical and stationary in one rest frame as in another. These considerations lead us to take the spherical approximation fairly seriously. The net effect of a collapsing texture is a net velocity impulse inward which is independent of radius in the inner regions (Turok and Spergel 1990). The magnitude of the impulse is

$$v_{tex} = \epsilon_t c \approx 9 \times 10^{-4} c = 270\,\mathrm{km s^{-1}} \tag{3.1}$$

where ϵ_t is determined by the parameters of the theory and the numerical value was taken from an estimate of the normalization by Gooding *et al.* (1991). This velocity impulse is two orders of magnitude larger than that expected from strings indicating that we expect much larger clumps for textures than for strings. The COBE DMR detection of CMB fluctuations (Smoot *et al.* 1992) is consistent with this value of ϵ provided that galaxy formation is sufficiently biased, $b = 3 \pm 1$ (Pen, Spergel and Turok 1992).

If we ignore self gravity we may solve the equation of motion for the comoving radius of a shell and the resulting overdensity

$$
r(t,t_i) = \left| r_i - \frac{v_{tex}\, t_i}{a_i} \ln \frac{t}{t_i} \right|
$$

$$
1+\delta(r,t,t_i) = \sum_{\rm streams} \frac{{r_i}^2}{r^2}\left|\frac{dr_i}{dr}\right| = \begin{cases} 2\dfrac{\left(\frac{v_{tex} t_i}{a_i}\ln\frac{t}{t_i}\right)^2 + r^2}{r^2} & r < \dfrac{v_{tex}\, t_i}{a_i}\ln\dfrac{t}{t_i} \\ \dfrac{\left(\frac{v_{tex} t_i}{a_i}\ln\frac{t}{t_i} + r\right)^2}{r^2} & r > \dfrac{v_{tex}\, t_i}{a_i}\ln\dfrac{t}{t_i} \end{cases} \tag{3.2}
$$

where we have summed over the incoming and outgoing stream of particles where they overlap. The r^{-2} divergence of the density is just a kinematical effect due to the assumption that all of the particles are on exactly radial orbits. The gravitational potential from this mass distribution near the center is

$$
\Phi(r,t) \approx \frac{3}{8}\Omega_{\rm cdm}{v_{tex}}^2\left(\frac{a}{a_i}\right)^2\left(\frac{t_i}{t}\ln\frac{t}{t_i}\right)^2 \ln r \approx \frac{3}{4}\frac{{a_i}^2}{a\,a_{eq}}{v_{tex}}^2\left(\ln\frac{a}{a_i}\right)^2 \ln r \qquad r \ll a_i v_{tex}\, t_i \ln\frac{t}{t_i} \tag{3.3}
$$

which diverges logarithmically near the center. We must assume that the singularity at the center gets softened at sufficiently small radii so that we do not produce a black hole! However this can be at a very small radius since the divergence is logarithmic. The fact that the potential diverges at the center says that the solution (3.2) is not really self-consistent. The large potential well depth will cause the particles to move much faster at the center than is indicated by equation (3.2). However, this does not necessarily indicate that the particle will virialize (i.e., start to oscillate around the center) after passing through the center. In order for that to occur the particles must loose a significant fraction of their energy as they pass through the center. The energy loss is given by

$$
\frac{\Delta E}{m} = \int \dot\Phi(r(t),t)dt = \int \dot\Phi(r,t)\frac{a(t)}{|v(t)|}\,dr \tag{3.4}
$$

which is to be compared to the kinetic energy per unit mass. According to equation (3.3), this is

$$
\frac{\text{K.E.}}{m} = \frac{1}{2}{v_{tex}}^2\left(\frac{a_i}{a}\right)^2. \tag{3.5}
$$

Clearly the logarithmic divergence of the potential is an integrable divergence in equation (3.4) and has no great effect. Comparing equation (3.3) and (3.5) we see that the effect of the self gravity of the CDM is negligible until $a \sim a_{eq}$ with some logarithmic corrections. This is just the usual result of linear theory. No virialization will occur during the radiation era. As in the case of strings the clumps will not virialize until z_{eq}.

The clumps which first collapse at z_{eq} will be much the same as in the string scenario, the biggest difference being that the velocity impulse which produces the clumps is $10^{-3}c$ rather than $10^{-5}c$. Scaling equations (2.4), (2.10), (2.11), and (2.12) we find

$$
\begin{aligned}
z_{\rm damp} &\sim \frac{\epsilon_t c}{v_{\rm cdm}(0)} \sim 5\times 10^7 \left(\frac{4g_{\rm dec}}{11}\right)^{\frac{1}{3}} \left(\frac{M_x}{1\,{\rm GeV}}\right)^{\frac{1}{2}} \left(\frac{T_{\rm dec}}{1\,{\rm MeV}}\right)^{\frac{1}{2}} \\
R_{\rm core} &\sim \epsilon_t c t_i \sqrt{\frac{t_{\rm eq}}{t_i}\ln\frac{t_{\rm eq}}{t_i}} \in 1\,{\rm pc}\left[1, \frac{z_{\rm eq}}{z_{\rm damp}}\ln\frac{z_{\rm damp}}{z_{\rm eq}}\right] \Omega_0^{-2}h^{-4} \\
M_{\rm core} &= \rho_{\rm eq} R_{\rm core}^3 \in 3\times 10^7\,{\rm M}_\odot \left[1, \left(\frac{z_{\rm eq}}{z_{\rm damp}}\ln\frac{z_{\rm damp}}{z_{\rm eq}}\right)^3\right] \Omega_0^{-2}h^{-4} \\
v_{\rm core} &= \sqrt{\frac{GM_{\rm core}}{R_{\rm core}}} \in 100\,{\rm km\,s^{-1}} \left[1, \left(\frac{z_{\rm eq}}{z_{\rm damp}}\ln\frac{z_{\rm damp}}{z_{\rm eq}}\right)^2\right]
\end{aligned}
\tag{3.6}
$$

To estimate the fraction of matter which is contained in these clump cores we can use that the rate of knot unwinding per unit time, per unit comoving volume in the radiation era is (Gooding *et al.* 1991)

$$
\frac{dn}{dV_{\rm co}dt_i} = k\frac{a_i{}^3}{c^3 t_i{}^4} \qquad k \sim 0.04. \tag{3.7}
$$

Thus the fraction of the volume accreted is

$$
f_{\rm nl,eq} \approx \int_{t_{\rm damp}}^{t_{\rm eq}} \frac{dn}{dV_{\rm co}dt_i}\frac{4\pi}{3}\left(\epsilon_t\frac{ct_i}{a_i}\ln\frac{t_{\rm eq}}{t_i}\right)^3 dt_i = \frac{16\pi}{3}k\epsilon_t^3\left[\ln\frac{z_{\rm damp}}{z_{\rm eq}}\right]^4 \sim 10^{-4}. \tag{3.8}
$$

In contrast to the case of strings, a very small fraction of the matter will virialize at matter-radiation equality. We also see from the above that the mass distribution of clump cores is

$$
n(M_{\rm core})\,dM_{\rm core} \overset{\propto}{\sim} \left[\ln\frac{3\times 10^7{\rm M}_\odot}{M_{\rm core}}\right]^3 \frac{dM_{\rm core}}{M_{\rm core}^2} \tag{3.9}
$$

which is similar to equation (2.13) for strings.

The kinematical $\rho \sim r^{-2}$ divergence encountered in the radiation era persists in the matter era if we continue to insist on radial infall. That this is generic for radial collapse including the effects of virialization has been shown by Fillmore and Goldreich (1984). This is important for us since the clumpiness of an object with such a profile diverges. Two effects will moderate this. One is that the CDM particles did not start out with zero velocity dispersion. The small initial thermal velocities will cause the particles to miss the center. Secondly, the outer regions of a collapsing texture is less spherical than the inner region and the impulse given to the particles will not be directly exactly toward the center. It is reasonable to assume that these two effects will prevent this kinematical divergence from persisting long after the texture is seeded. Outside the core region which collapses at $z_{\rm eq}$ (i.e., for $r \gg \frac{v_{\rm tex}t_i}{a_i}\ln\frac{t_{\rm eq}}{t_i}$), the density profile falls off as

r^{-1}. Purely radial infall will give it the $\rho \sim r^{-2}$ already encountered when it virializes. On the other hand, if the infall is highly non-radial or highly non-spherical then particles will (roughly) maintain their radial ordering after they virialize, leading to a $\rho \sim r^{-\frac{3}{2}}$ density profile. The clumpiness of such an outer $r^{-\frac{3}{2}}$ halo grows logarithmically with radius. Thus to understand the clumpiness we must understand the outer halo of the clumps.

Clumps seeded by textures will accrete a much more substantial halo than string-seeded halos. This is because the fraction of matter in clumps is quite small at $z_{\rm eq}$ and because the clumps are isolated unlike fragments of a string accretion wake. Thus there is not much competition between clumps, and the $\rho \sim r^{-\frac{3}{2}}$ halo mentioned above has a chance to come into existence. As indicated in Gooding *et al.* (1991) the r^{-1} linear density profile induced by a collapsing knot is truncated at the comoving horizon when the knot was seeded, i.e., $ct_{\rm i}/a_{\rm i}$. Clumps seeded in the radiation era will accrete up to their truncation radius at a redshift

$$z_{\rm acc} = \epsilon_{\rm t} z_{\rm eq} \ln\frac{t_{\rm eq}}{t_{\rm i}} \sim 10 \ln\frac{t_{\rm eq}}{t_{\rm i}}. \tag{3.10}$$

Accretion after $z_{\rm acc}$ will lead to a much steeper density profile and will not lead to increasing clumpiness. At $z_{\rm acc}$ the physical accretion radius will be a factor of $\left(\epsilon_{\rm t}\ln\frac{t_{\rm eq}}{t_{\rm i}}\right)^{-2}$ times larger. It is the logarithm of this factor which gives the additional clumpiness from the outer halo, a factor of 14 for the largest clumps to a factor of 8 for the smallest. Here we will take a correction factor of 10. Our estimate of the clumping factor is thus

$$C = \overline{\delta^2_{\rm cdm}} \sim 10 f_{\rm nl,eq} f_{\rm cl} {z_{\rm eq}}^3 \sim 10^9 f_{\rm cl} h^6 \Omega_0^3 \tag{3.11}$$

where as above $f_{\rm cl}$ is the survival fraction of the clumps.

We should mention that the masses of the halos of the clumps are much larger than that of the cores, because for a r^{-1} linear density profile the comoving accretion radius grows as the expansion factor, and thus the mass accreted grows as the cube of the expansion factor. Combining (3.6) and (3.10) we find

$$\begin{aligned}
R_{\rm halo} &\sim \frac{ct_{\rm i}\sqrt{\frac{t_{\rm eq}}{t_{\rm i}}}}{\epsilon_{\rm t}\ln\frac{t_{\rm eq}}{t_{\rm i}}} \in 1\,{\rm Mpc}\left[1, \frac{z_{\rm eq}}{z_{\rm damp}}\left(\ln\frac{z_{\rm damp}}{z_{\rm eq}}\right)^{-1}\right] \\
M_{\rm halo} &= \left(\frac{z_{\rm acc}}{z_{\rm eq}}\right)^3 \rho_{\rm eq} R^3_{\rm halo} = \rho_{\rm eq}(c\sqrt{t_{\rm i}t_{\rm eq}})^3 \sim 3\times 10^{16}\,{\rm M}_\odot \left[1, \left(\frac{z_{\rm eq}}{z_{\rm damp}}\right)^3\right]. \\
v_{\rm halo} &= \sqrt{\frac{GM_{\rm halo}}{R_{\rm halo}}} \sim 10^4\,{\rm km\,s^{-1}}\left[1, \frac{z_{\rm eq}}{z_{\rm damp}}\sqrt{\ln\frac{z_{\rm damp}}{z_{\rm eq}}}\right]
\end{aligned} \tag{3.12}$$

Note that $M_{\rm halo}$ is independent of $\epsilon_{\rm t}$. Of course, in this scenario these halos are actually the halos of galaxies and clusters (Turok 1989). The significance of CDM clumps in the texture scenario is quite different than clumps in the other scenario for this reason. There is really no difference between the clumps and other bound astronomical objects. Galaxies and galaxy clusters are just the large mass end of the clump distribution.

IV. INFLATION

The canonical CDM model is the residue of an inflationary epoch that sets $\Omega = 1$ and generates a Zel'dovich a Gaussian distribution of primordial curvature fluctuations. The spectrum approaches the $n = -3$ limit on subgalactic mass scales, where non-linearity occurs almost simultaneously at a redshift $1 + z_{\rm nl} \sim 30/b$, where the bias factor b is the inverse of the rms fluctuation amplitude in the dark matter on the $8h^{-1}$Mpc scale where fluctuations in the galaxy counts have unit amplitude. Galaxy peculiar velocities and the correlation function slope, on small scales, and large scale drift velocities, clusters, superclusters and voids on large-scales, constrain b to lie in the range $1.5 \lesssim b \lesssim 2.5$. The issue of whether there is a unique value of b which simultaneously satisfies all constraints is controversial, and is not directly relevant to our discussion. We henceforward adopt a value $b = 2$ for illustrative purposes.

Mass scales condense hierarchically according to the cold dark matter prescription for the fluctuation power spectrum $|\delta_k|^2 \propto k^{n_{\rm eff}}$ where the effective spectral index $n_{\rm eff}$ is a function of mass scale M. The time at which a fluctuation containing mass M begins to collapse is given by $t \propto M^{\alpha}$, where $\alpha = (n_{\rm eff} + 3)/4$, with α and $n_{\rm eff}$ varying from 0.12 and -2.5, respectively, at $10^6 M_\odot$ to 0.2 and -2.2, respectively, at $10^{10} M_\odot$. We shall assume that sufficiently rare and isolated fluctuations, at say the 2σ level, are able to undergo collapse until nonlinearity on larger mass scales incorporates them into larger mass–scales. We distinguish successive scales by a factor 2 in filter scale, since 2σ fluctuations are approximately a factor 2 smaller in mean diameter than the rms filter scale.

For survival, we must require a large density contrast to have developed. If the smaller scale fluctuations have a density contrast of ~ 100, they should be able to survive tidal disruption or dynamical friction drag when its environment undergoes non-linear collapse. This condition is satisfied, for a relative ratio of filter scales of 2:1 as appropriate to a 2σ fluctuation, provided that $\alpha \gtrsim 0.15$. This translates into a minimum mass scale of $\sim 10^8 M_\odot$ for surviving substructure. A more refined argument is given by Silk and Szalay (1987), but for the present purpose, it suffices to note that the clumpiness factor is sensitive, not to the minimum mass scale, but to the density at which non-linearity occurs and to the mass fraction f in surviving clumps. We estimate that $f \sim 0.01$, with ~ 10 percent of the mass being present in $\sim 2\sigma$ fluctuations and ~ 10 percent of the mass in these fluctuations at turn-around being trapped in the cores at a density enhancement of $\delta_{\rm cl}$ of ~ 100, relative to the turn-around density, this latter estimate assuming a $\rho \overset{\propto}{\sim} r^{-2}$ density profile. Our inferred clumpiness factor is therefore

$$C = f\frac{5.5\rho(z_{nl})}{\rho_0}\delta_{\rm cl} = 2 \times 10^4 \frac{f}{0.01}\frac{\delta_{\rm cl}}{100}\left[\frac{1+z_{\rm nl}}{15}\right]^3 \left[\frac{b}{2}\right]^3 \tag{4.1}$$

V. THE FATE OF CLUMPS

We have established that, in the scenarios considered above, the CDM will be very clumpy at early times. Crucial issues are whether the clumps of CDM will survive until today and where they will be located. First consider survival in scenarios in which the CDM is seeded. We see from equations (2.13) and (3.10) that in the string and texture scenario most of the clumped mass is in the very smallest clumps (by a logarithmic factor). Thus what happens to the small clumps is the most important issue. The characteristics of these clumps are given by the lower limits of equations (2.10-12) and (3.6), i.e.

$$
\begin{aligned}
R_{\rm core} &= v_{\rm cdm}(0)\, t_{\rm eq}\, z_{\rm eq} \ln\left[\frac{\epsilon c}{z_{\rm eq} v_{\rm cdm}(0)}\right] \sim 0.001\,{\rm pc}\,\mathcal{L}\left(\frac{11}{4g_{\rm dec}}\right)^{\frac{1}{3}}\left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{1}{2}}\left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{1}{2}}\Omega_0^{-1}h^{-2} \\
M_{\rm core} &= \rho_{\rm eq} R_{\rm core}^3 \sim 0.02\,{\rm M}_\odot\,\mathcal{L}^3\left(\frac{11}{4g_{\rm dec}}\right)\left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{3}{2}}\left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{3}{2}}\Omega_0^{-1}h^{-2} \\
v_{\rm core} &= \sqrt{\frac{GM_{\rm core}}{R_{\rm core}}} \sim 0.2\,{\rm km\,s^{-1}}\,\mathcal{L}\left(\frac{11}{4g_{\rm dec}}\right)^{\frac{1}{3}}\left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{1}{2}}\left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{1}{2}}
\end{aligned}
\tag{5.1}
$$

where ϵ is ϵ_s or ϵ_t for strings or textures, respectively, and $\mathcal{L}$ is a logarithmic correction factor given by

$$
\mathcal{L} = 1 + \frac{\ln\frac{\epsilon}{z_{\rm eq} v_{\rm cdm}(0)}}{\ln\frac{10^{-6}}{(2.5\times10^4)(10^{-11})}} = 1 + 0.27\ln\left[\left(\frac{\epsilon}{10^{-6}}\right)\left(\frac{4g_{\rm dec}}{11}\right)^{\frac{1}{3}}\left(\frac{M_x}{1\,{\rm GeV}}\right)^{\frac{1}{2}}\left(\frac{T_{\rm dec}}{1\,{\rm MeV}}\right)^{\frac{1}{2}}\Omega_0^{-1}h^{-2}\right].
\tag{5.2}
$$

Note that the value of ϵ does not enter except through the logarithmic correction term, thus it makes little difference whether it is strings or textures which we are talking about. This is easy to understand. The size of the seeded objects is equal to the velocity boost given by the string or texture. The smallest object produced are those for which the velocity boost is equal to the velocity dispersion. Thus the comoving size of the smallest objects is just the free-streaming length of the CDM particles. This free-streaming scale only grows logarithmically with time in the radiation era. Since $z_{\rm dec}$ is proportional to ϵ we have a logarithmic dependence on ϵ. The size and mass of these smallest clumps is thus generic. Any theory which induces sufficient inhomogeneities on small scales will have produce nonlinear objects in the radiation era with a small mass cutoff similar to that estimated here. The reason Gaussian initial conditions with a Harrison-Zel'dovich spectrum does not is because there is not sufficient power on small scales and the Gaussian distribution makes fluctuations much larger than the rms very rare. The reason that strings and textures do produce these small clumps is partly due to their non-Gaussian nature in which the large fluctuations in the form of accretion wakes of strings or collapsing texture knots are relatively common.

The cores of the smallest CDM clumps are very similar whether seeded by strings or by textures. This is not the case for the halos. We have argued that in the string scenario accretion is highly suppressed, the halos being ~ 10 times more massive than the cores. In the texture seeded scenario the situation is reversed with the accretion factor much greater than z_{eq}. From equation (3.12) we see that even the smallest clumps accrete halos with characteristics

$$
\begin{aligned}
R_{\rm halo} &\sim \frac{z_{\rm eq} v_{\rm cdm}(0) t_{\rm eq}}{\epsilon_{\rm t}^2} \ln\left[\frac{\epsilon_{\rm t} c}{z_{\rm eq} v_{\rm cdm(0)}}\right] \sim 3\,{\rm kpc}\,\mathcal{L} \left(\frac{11}{4g_{\rm dec}}\right)^{\frac{1}{3}} \left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{1}{2}} \left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{1}{2}} \Omega_{\rm o}^{-1} h^{-2} \\
M_{\rm halo} &\sim \rho_{\rm eq}\left(\frac{z_{\rm eq}}{\epsilon_{\rm t}} v_{\rm cdm}(0) t_{\rm eq}\right)^3 \sim 10^5\,{\rm M}_\odot \left(\frac{11}{4g_{\rm dec}}\right) \left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{3}{2}} \left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{3}{2}} \Omega_{\rm o}^{-1} h^{-2} \\
v_{\rm core} &\sim 0.4\,{\rm km\,s^{-1}}\,\mathcal{L}^{-\frac{1}{2}} \left(\frac{11}{4g_{\rm dec}}\right)^{\frac{1}{3}} \left(\frac{1\,{\rm GeV}}{M_x}\right)^{\frac{1}{2}} \left(\frac{1\,{\rm MeV}}{T_{\rm dec}}\right)^{\frac{1}{2}}
\end{aligned}
\tag{5.3}
$$

where the logarithmic correction factor, $\mathcal{L}$, given by

$$
\mathcal{L} = 1 + \frac{\ln\frac{\epsilon}{z_{\rm eq} v_{\rm cdm}(0)}}{\ln\frac{10^{-3}}{(2.5\times10^4)(10^{-11})}} = 1 + 0.12 \ln\left[\left(\frac{\epsilon}{10^{-3}}\right) \left(\frac{4g_{\rm dec}}{11}\right)^{\frac{1}{3}} \left(\frac{M_x}{1\,{\rm GeV}}\right)^{\frac{1}{2}} \left(\frac{T_{\rm dec}}{1\,{\rm MeV}}\right)^{\frac{1}{2}} \Omega_{\rm o}^{-1} h^{-2}\right]
\tag{5.4}
$$

The mass of the halos may determine the ultimate fate of the cores of the clumps.

A crucial stage for the evolution of the CDM clumps formed at z_{eq} is the epoch when they start to interact with neighboring clumps. In the string scenario, clumps are right next to their nearest neighbor clumps and will start to cluster and merge soon after z_{eq}. In the texture scenario, the collapsing knots are isolated but will rapidly start to accrete a large halo. They will not start to interact with their neighbors until well into the matter era. Note that it is the halos which are most important for the interaction, not the dense cores. The type of interaction may take several forms. One clump can accrete another clump into their halos. If the accreted clumps have sufficiently different masses then dynamical friction will be ineffectual in dragging smaller clump in toward the core of the other. Thus the two may coexist, one in the halo of the other. On the other hand if the two clumps have similar enough masses then dynamical friction will cause the two to spiral toward each other forming a single undifferentiated object. In other words, the two clumps will merge. In the string or texture scenario, we are most interested in whether the smallest clumps will merge with each other since they have most of the clumpiness. Merging of small clumps can be shut off if they are accreted by much larger objects, since the two smaller clumps will be tidally separated by the gravitational field of the larger object. This can be an effective means of preserving the small clumps because in these scenarios perturbations on a large range of scales exist simultaneously. The largest objects grow by accreting the surrounding medium, including much smaller bound object, and not by mergers with similarly sized objects. The structures in the string or texture scenario grow predominantly by secondary infall rather than hierarchical clustering and merging. Of course at late times, $1 + z \sim 1$, if the structures seeded around z_{eq} have accreted most of the matter in the universe then merging may become important, but on a scale much larger than the clumps.

Just how much of the clumpiness might be removed by the initial merging stages is not totally clear. If the merger remnant retains a dense core then the clumpiness may not be significantly effected by this process at all. In any case there should be a significant fraction of clumps which do not merge. Determining the effective survival fraction, $f_{\rm cl}$, from this early merging epoch requires more study, however we feel that values $f_{\rm cl} \gtrsim 0.1$ are quite plausible in the string scenario and $f_{\rm cl} \sim 1$ are quite likely in the texture scenario.

In the inflationary scenario, the clumps are much less dense and considerably more massive than both the smallest clumps in the texture scenario and the largest clumps in the string scenario. They will also be subject to a period of potential merging soon after they form. The clumps, being 2σ peaks of a Gaussian random field filtered at the $10^7\,\mathrm{M}_\odot$ scale, will tend to lie in high density regions filtered on a much larger mass scale, i.e., they are biased (Bardeen *et al.* 1986). Thus the clumps will tend to be more bound to their nearby neighbors than if they were distributed randomly. This may make them more subject to mergers in some cases, but may also prevent mergers by placing them in a high velocity dispersion environment. We note that galaxies, which also may have been formed from 2-σ peaks of a Gaussian random density field, may retain their identity for long periods of time in the high velocity dispersion environment inside of a rich cluster of galaxies. As with seeded clumps it is difficult to accurately estimate the fraction of dense clumps which survive the epoch of mergers. Here we suggest that $f_{\rm cl} \sim 0.1$ is a plausible value.

Now consider survival of the clumps at the present epoch. In the string or texture scenario, most of the mass in dense clumps is in the very smallest clumps which were seeded at very early times. These clumps are thus unrelated to the perturbations which formed the large scale structure in our universe. Therefore we do expect to find many clumps of CDM between the galaxies, since most of the matter in the universe has not accreted onto galaxies. However even those clumps which do fall into galaxies are fairly robust. The central cores of the seeded clumps have a density equal to the density of CDM at the epoch of matter-radiation equality, i.e., roughly $10^7\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}\Omega_0^4 h^8$. Aside from compact objects (stars and planets) and molecular cloud cores, these cloudlets are denser than any other known astronomical object, at least if $\Omega_0 \sim 1$. In particular they are denser than galaxy clusters, galaxies, and other smaller star clusters. Thus while the outer halos of these clumps may be tidally stripped through interactions with these objects the cores will not be tidally disrupted. A similar argument can be made for clumps in the inflationary scenario even though their density is much less.

Clumps which are not tidally disrupted by a galaxy may still spiral into the center due to dynamical friction. This is much more likely to happen in smaller dwarf galaxies than in large bright galaxies. Here we will concentrate on galaxies like our own. For a given clump, it is the mass of the outer halo of the clump and not the inner core which will determine the dynamical friction time, which is inversely proportional to the mass of the clumps. Something as massive as a globular cluster ($\sim 10^6\mathrm{M}_\odot$) will be drawn into the center from a distance of $\sim$ 1kpc in about a Hubble time (Binney and Tremaine 1987). In the string-seeded case, we see that not even the largest clumps, and including any halo, would be massive enough to be drawn into the Galactic center by dynamical friction. In contrast in the inflationary scenario essentially all clumps in the inner parts of the Galaxy may have be drawn toward the center.

When considering dynamical friction and texture seeded clumps, we should note that much of the halo of any texture seeded clump will be tidally stripped by our Galaxy. Nevertheless we can be assured that some of the larger clumps will retain enough mass so that dynamical friction will be able to drag the clumps to the center. At the high mass end this process will be what is normally referred to as galaxy merging. We can also be assured that the smallest clumps can escape dynamical friction in the inner regions of our galaxy, and in particular, that clumps will remain at the solar galactic radius. An important difference between the texture scenario and the other scenarios is that our Galaxy is probably just a large clump and should have a dense core of CDM at its center. Even if the Galaxy were the merger of a few large clumps this conclusion would remain. This does not mean that the Galaxy is dominated by CDM all the way to its center, as baryonic infall could change this. There should however be a core of CDM with a density $\sim \rho_{eq}$ at the center. Comparing equations (3.12) and (3.6) we see that if we assign a mass of $10^{12}M_\odot$ to the Galactic halo then the mass in the core should be $\sim 10^6 M_\odot$. It is quite plausible that cooling baryons might form a black hole in the center which could subsequently swallow some of the central core of CDM. However since the CDM has no mechanism to lose angular momentum, it is unlikely that the black hole would swallow much of the CDM core.

In addition to being eaten by larger objects, clumps may also destroy themselves. There are at least two mechanisms by which clumps could decrease their density. One is through collisions with other clumps or stars, in which the gravitational interaction will "heat" the clump core causing its density to decrease. It is the head-on rather than the large impact parameter collisions which do the most damage. Since the velocity of the clumps in a galaxy halo is much greater than their internal velocity dispersion, many head-on collisions will be required to have a significant effect. We do not expect collisions will have much of an effect on the cores of the CDM clumps in the Galaxy in any of the scenarios.

The other mechanism by which CDM clumps can decrease their density is through mass loss by annihilating CDM particles. This clearly would not apply to axions which do not annihilate. According to the usual calculation of the freeze-out of particle number density, the annihilation cross-section at freeze-out is given by

$$\langle \sigma v \rangle_{\rm freeze} \approx 3 \times 10^{-27} {\rm cm^3 s^{-1}} (\Omega_0 h^2)^{-1} \left(\frac{88}{g_{\rm freeze}} \right)^{\frac{1}{2}} \left(\frac{M_x c^2}{25 T_{\rm freeze}} \right) \tag{5.5}$$

(Kolb and Turner 1989), where $g_{\rm freeze}$ is the effective number of relativistic degrees of freedom at freeze-out. We may write the present annihilation rate as

$$\langle \sigma v \rangle = c_\sigma \langle \sigma v \rangle_{\rm freeze} \tag{5.6}$$

If s-wave annihilations dominate at freeze out then $c_\sigma \sim 1$ while in general $c_\sigma < 1$. Today the rate of density loss in a clump directly to annihilation is

$$\dot{\rho} \sim M_x n_x^2 \langle \sigma v \rangle \qquad \longrightarrow \qquad H_0^{-1}\frac{\dot{\rho}}{\rho} = 0.3\, c_\sigma \frac{\rho}{\rho_{\rm eq}} \frac{1\,{\rm GeV}}{M_x} \Omega_0^3 h^5. \tag{5.7}$$

It is clear that in the inflationary scenario annihilations cannot have much of a dynamical effect since $\rho_{\rm core} \ll \rho_{\rm eq}$. On the other hand, for the texture and string models annihilation may just be beginning to be important for the lighter dark matter candidates ($M_x \sim 1\,{\rm GeV}$). Let us consider the dynamical effects of annihilation. Suppose the core of a clump initially has density $\rho_{\rm i}$ then we may define an annihilation timescale

$$\frac{1}{\tau} \equiv \frac{\dot{\rho}_{\rm i}}{\rho_{\rm i}} = \frac{\rho_{\rm i}}{M_x} \langle \sigma v \rangle. \tag{5.8}$$

We will, of course, assume that all of the annihilation products leave the core of the clump. Since the annihilation timescale is much longer than the dynamical timescale, as the clump core loses mass the orbits or the remaining particles will adiabatically expand. The adiabatic invariant is just the size of the orbit times the velocity of the particle. Using this one may show

$$M_{\rm core} = \frac{M_{\rm core,i}}{(1 + 4\frac{t}{\tau})^{\frac{1}{4}}} \qquad \rho_{\rm core} = \frac{\rho_{\rm core,i}}{1 + 4\frac{t}{\tau}}. \tag{5.9}$$

The clumpiness of the medium will thus decay as $(1 + 4\frac{t}{\tau})^{-\frac{5}{4}}$. However we see from (5.7) that this effect will only be important for the lightest CDM candidates.

We conclude this section by noting that one of the largest uncertainties concerns the survival fraction of clumps. To this we can only offer educated guesses. Even a very small survival fraction can lead to significant clumpiness. Thus the difficult question that must be answered is just how rare is the special environment required for the survival of clumps. This is clearly an area which requires further study before we can decide just how much clumpiness we should expect.

VI. OBSERVABLE IMPLICATIONS

For many purposes it does not matter whether the dark matter is clumped or smoothly distributed. The dynamical effect on galaxy rotation curves, galaxy cluster velocity dispersion, or large scale peculiar velocities will not depend on whether the dark matter is clumped or smoothly distributed within the halos. If the halo of our Galaxy is too lumpy then heating of the observed thin disk may occur. However in none of the scenarios is this likely to be much of a problem. Even if all of the halo were made of clumps, the clumps would have to have a mass greater than $10^6 M_\odot$ for unacceptable heating to occur (Lacey and Ostriker 1985). In the string or texture scenario the smallest clumps which have most of the mass are smaller than $10^6\, M_\odot$. In all of the scenarios the larger clumps which might cause a problem would contain only a small fraction of the mass of the halo.

Another way one might observe these clumps is if they were to accrete a baryonic halo. First consider the smallest clumps in either the string or texture scenario. From equation (5.1) we see that the virial temperature of the cores of the smallest clumps is only $\sim 10\,\mathrm{K}$. It is unlikely that any significant amount of baryons achieved such a low temperature so we do not expect them to accrete baryons. The halos of the small clumps in the string scenario will have even lower virial temperatures so they also will not accrete baryons. We see from equation (5.3) the halos of the smallest clumps in the texture scenario have only marginally greater virial temperature than the cores so again there will be no baryonic accretion.

In the inflationary CDM scenario where the clumps are about the size and mass of globular clusters, we might plausibly expect baryonic accretion. In fact these clumps might be the origin of globular clusters. If so then the CDM concentrations in cores of these globular clusters can affect stellar evolution. A plausible range of cross-section allows WIMP energy transport to result in the development of an isothermal core in main sequence stars. This can affect the observed luminosity function of globular cluster stars (Dearborn, *et al.* al. 1992).

In any case, dynamical friction will cause some of the population of globular cluster-sized clumps to spiral into the disk, resulting in a source of disk dark matter. (Lake 1989), and also to spiral into galactic nuclei, leading to dark matter in the central spheroids. One feature of inhomogeneous dissipationless collapse is that clumps tend to preserve their initial density, despite undergoing considerable merging. One therefore ends up with a central cloud of CDM, whose density is similar to the cold dark matter density in clumps which made it, i.e. $\lesssim 10\,\mathrm{M_\odot pc^{-3}}$. As mentioned previously, in the texture scenario a central cloud of CDM is expected to form before the galaxy. One observational signature of these central CDM clouds include detection of a concentrated dark matter core in a galaxy whose nucleus is apparently too diffuse to allow black hole formation by conventional means (Salati and Silk 1989).

CDM annihilations can result in a detectable diffuse background of gamma rays. Annihilations of a majorana particle candidate for CDM, such as the photino, result in GeV gamma rays, as well as energetic neutrinos, electron-positron pairs, and proton-antiproton pairs (Silk and Srednicki 1984). One can also observe the resulting isotropic γ-ray flux produced by CDM annihilations at characteristic energy $\sim 0.1 m_x$ back to $z \sim 100$ prior to which the universe absorbed GeV gammas via pair production, $\gamma + p = \gamma + \bar{p} + e^+ + e^-$ (Stecker 1978). In the case of annihilation of CDM in fixed density clumps, most of the emission comes from the last expansion time. Let us parameterize the fraction of the annihilation energy which is emitted as energetic photons by b_γ. Most the energy released in annihilations will have been emitted in the last Hubble time so the present energy density annihilation photons will be

$$\mathcal{E}_\gamma \sim b_\gamma H_0^{-1} C \frac{\rho_0^2 c^2}{M_x} \langle\sigma v\rangle \sim 1.5\times 10^{-22} C\, b_\gamma\, c_\sigma\, \Omega_0\, h \left(\frac{1\,\mathrm{GeV}}{M_x}\right) \left(\frac{88}{g_{\mathrm{freeze}}}\right)^{\frac{1}{2}} \left(\frac{M_x c^2}{25 T_{\mathrm{freeze}}}\right) \mathrm{erg\,cm^{-3}}, \tag{6.1}$$

where C is the clumpiness factor and we have used equation (5.5) for the cross section. The energy at which most annihilation photons will be emitted will depend on details of the annihilation channels, but generically we might expect $E_\gamma \sim 0.1 M_x c^2$. It is easy to see that in the inflationary scenario the clumpiness factor is too small to produce a significant γ-ray background compared to what is observed. For strings or textures where the clumps have a density $\sim \rho_{\mathrm{eq}}$ we see that

$$\mathcal{E}_\gamma \sim 2.3 \times 10^{-9} f_{\mathrm{nl}}\, f_{\mathrm{cl}}\, b_\gamma\, c_\sigma\, \Omega_0^5\, h^9 \left(\frac{1\,\mathrm{GeV}}{M_x}\right) \left(\frac{88}{g_{\mathrm{freeze}}}\right)^{\frac{1}{2}} \left(\frac{M_x c^2}{25 T_{\mathrm{freeze}}}\right) \mathrm{erg\,cm^{-3}}. \tag{6.2}$$

Observations clearly preclude that $f_{\mathrm{nl}}\, f_{\mathrm{cl}}\, b_\gamma\, c_\sigma\, \Omega_0^5\, h^9 \sim 1$, but a small survival fraction with a low Hubble constant is possible.

We may not only detect the general background of radiation produced by annihilating matter in clumps, but it is possible to see individual clumps as well. Again we restrict our attention to the dense clumps produced in the string or texture scenario. Let us consider the smallest clumps which produce most of the γ-rays. The dark-matter halo density at the solar radius is $\rho_{\mathrm{h}} \sim 10^{-2}\,\mathrm{M_\odot\,Mpc^{-3}}$. Given that a fraction $f_{\mathrm{nl}} f_{\mathrm{cl}}$ of the dark matter in these smallest clumps, the angular size of the nearest clump should be

$$\theta_{\mathrm{n}} \sim \left(f_{\mathrm{nl}} f_{\mathrm{cl}} \frac{\rho_{\mathrm{h}}}{\rho_{\mathrm{eq}}}\right)^{\frac{1}{3}} \sim 5' \left(f_{\mathrm{nl}} f_{\mathrm{cl}}\, \Omega_0^{-4} h^{-8}\right)^{\frac{1}{3}} \tag{6.3}$$

which certainly would be unresolved by any planned γ-ray observatories. With the mass function of clumps given by equations (2.13) or (3.9) the nearest small clump should have a larger angular size (by a logarithmic factor) than the nearest big clump. The number density of the bigger clumps are so much smaller that the nearest ones tend to be much further away. The central bolometric γ-ray brightness of the string- or texture-seeded clumps is ρ_{eq} is given by

$$\begin{aligned} B_\gamma \sim & \frac{\rho_{\mathrm{eq}}^2}{3\pi M_x} \langle\sigma v\rangle R_{\mathrm{cl}} \\ \sim & 2 \times 10^{-15} b_\gamma c_\sigma \Omega_0^3 h^6 \left(\frac{88}{g_{\mathrm{freeze}}}\right)^{\frac{1}{2}} \left(\frac{M_x c^2}{25 T_{\mathrm{freeze}}}\right) \left(\frac{R_{\mathrm{cl}}}{1\,\mathrm{pc}}\right) \left(\frac{1\,\mathrm{GeV}}{M_x}\right) \mathrm{erg\,cm^{-2}s^{-1}sr^{-1}}, \end{aligned} \tag{6.4}$$

which would be difficult to detect even if it subtended a far greater solid angle. If f_{gal} is the CDM fraction of mass within the solar radius of our Galaxy, then the total flux from the inner Galaxy is down by a factor $f_{\mathrm{gal}}(\rho_{\mathrm{halo}} R_{\mathrm{gal}})/(\rho_{\mathrm{eq}} R_{\mathrm{cl}}) \sim 10^{-9} f_{\mathrm{gal}}$ when compared to the clump cores. Such a low flux would be difficult to detect. In the inflationary scenario this flux would be down by at least an additional factor of $\rho_{cl}/\rho_{eq} \sim 10^{-9}$. Thus while there may be enhanced emission from the Galactic center (Silk and Bloemen 1987) and/or the central halo (Lake 1990a,b), the flux should be fairly small.

There are many experimental efforts underway to detect the the Galactic halo dark matter directly, as it passes through the laboratory. The predicted rates for detection events usually assume the halo material is smoothly distributed in the halo, and thus a laboratory will, over the duration of the experiment, contain a fair sample of dark matter particles. If the halo is clumpy this will not be the case. When the earth is passing through a cloud of dark matter then the number of particles in the laboratory will be significantly greater than average, and while the earth is not the signal could be significantly less than average. If all the dark matter were in clouds as dense as ρ_{eq} then this would make dark matter detection untenable, since only a fraction $\rho_{halo}/\rho_{eq} \sim 10^{-9}$ of the time would the earth be passing through a dark matter cloud. However we do not really expect this to be the case. In all of the scenarios considered here, the small clouds will be tidally stripped of their outer halos as they accrete onto larger and larger structures. Some of this tidally stripped dark matter would eventually find itself to be more or less smoothly distributed throughout the Galactic halo. We do expect that a large fraction of the dark matter will be tidally stripped off the outer parts of the clump halos, and hence the density of dark matter in the laboratory will be close to the mean for the halo most of the time. However if the earth were passing through a cloud of dark matter then the signal might be significantly enhanced. In the rare circumstance that the earth would be passing through the core of a texture- or string-seeded clump then the enhancement might be as much as a factor of 10^9.

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